Dynamics of growing interfaces in porous media with viscous fingering

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We introduce a model of a growing interface within a random porous medium in the framework of solid-on-solid restrictions. The possibility of having viscous fingering effects is included through a height dependent probability for the advance of the interface. The inclusion of viscous fingering effects is regulated by a parameter. The temporal scaling of the interface seems to be unaffected but the spatial scaling varies with the intensity of the fingering. In principle, although the fingering effect could always be present in real interfaces, in porous media its intensity is very difficult to estimate.

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Recently, the scaling of driven interfaces has been the subject of extensive studies [1]. When a fluid invades a porous medium starting from a flat interface, an out-of-equilibrium self-affine rough interface is generated. The interface has been characterized through the scaling of the interfacial width with time and lateral size. The result is the determination of two exponents β and α describing the time dependent and static scaling of the saturation width, respectively.

The self-affine interface has a characteristic scaling function that takes the form

$$w(l,t)=L^{\alpha}f(t/L^{\alpha/\beta}) , \tag{1}$$

α being the roughness exponent because $w \sim L^{\alpha}$ when $t \gg L^{\alpha/\beta}$ and β the dynamical exponent because $w \sim t^{\beta}$ when $t \ll L^{\alpha/\beta}$.

Various models have been proposed in order to explain the experimental measured values for the exponents that characterize the interface [2–4]. Amaral *et al.* considered in particular, the influence of a gradient in the density of pinning sites of the porous medium.

None of the proposed models takes into account the viscosity of the invading fluid although recently, a non-solid-on-solid (non-SOS) model has been presented and the characteristics of the interface are studied for different viscosity ratios [5].

We present here a SOS model very similar to the Tang *et al.* [2] model, but we introduce the possibility of having fingering effects through a height dependent growing probability. In our model, the interface growth proceeds in a square lattice $L \times H$ sites in which a random number uniformly distributed among 0 and 1 is assigned to each site in order to represent the porous media opposition. The random numbers are representative of the quenched noise of the porous media which will be a function of the interface height but not of the time. Growth proceeds in a strip geometry with periodic boundary conditions in the directions parallel to the interface.

At the beginning, all columns are assumed to have the same height equal to zero. During the growth one column is randomly selected. The neighbor column whose height is lower than the selected column by two or more units advances one. If none of the neighbor grows, the selected column advances one unit provided that its pressure is greater than the quenched noise of the site just above. Otherwise no action takes place.

The concept of fingering in our model is included through a height dependent pressure. It is well known that the fingering phenomenon takes place through the instabilization of flat interfaces between two viscous fluids [6]. Roughly speaking, if in a flat interface a small perturbation appears the pressure gradient increases at the top of the perturbation. Then, the top of the perturbation begins to move faster than the rest of the interface. Hence the perturbation gets further ahead, which in turn increases the pressure gradient. The entire perturbation is, then, destabilized by the motion of the interface.

In order to include in our simulation the above described phenomenon we express the pressure as

$$P(h,\gamma)=P_0+\gamma|h(x,t)-\langle h(x,t)\rangle| , \tag{2}$$

where $P(h,\gamma)$ is the pressure in the selected column at the height $h(x,t)$, P_0 is the initial pressure equal to 0.66 in the simulations, γ is the coefficient which regulates the fingering, $h(x,t)$ is the height of the chosen column, and $\langle h(x,t)\rangle$ is the average height of the interface.

In the simulations we used system sizes of $L=1000$ and the results are averaged over 100 runs typically. The temporal roughness scaling and the spatial roughness are determined. The average height of all columns is representative of time.

In Fig. 1 several growing shapes for different values of the γ coefficients are shown. The fingering effect becomes evident as the γ coefficient increases. The shapes are different from the standard viscous fingering phenomena due to the neighbor column interactions included in our model.

The behavior of the surface width as a function of time can be seen in Fig. 2 for six different values of the γ coefficient between 0 and 10^{-3}. For $\gamma=0$, 1×10^{-4}, and 2×10^{-4} the simulations were made up to $\ln t=8$ because the pressure is very close to the critical value and the interface pins very often.

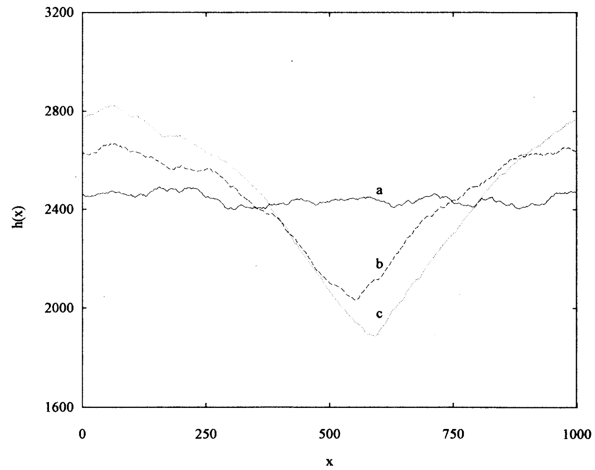

FIG. 1. Growing shapes for (a) $\gamma=0$, (b) $\gamma=3\times10^{-4}$, and (c) $\gamma=4\times10^{-4}$.

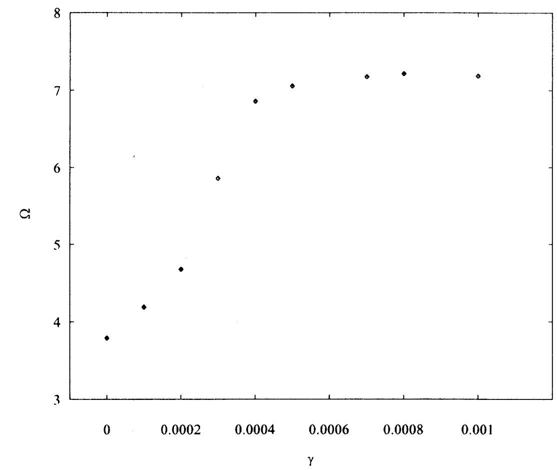

FIG. 3. Logarithm of the roughness value at saturation Ω as a function of the parameter γ.

For $\gamma=0$ no fingering effect takes place and the temporal scaling exponent results $\beta\cong0.82$. This value approaches very well previously reported results of a related model [2]. The saturation zone is well defined for the $\gamma=0$ case.

It can be seen that as the γ coefficient increases the temporal scaling exponent β remains the same, at least for the first part of the growing, but the width value at saturation increases. In Fig. 3 the logarithm of the saturation width (Ω) as a function of γ is plotted for γ between 0 and 10^{-3}. The saturation width increases initially very fast with γ, but at $\gamma\approx7\times10^{-4}$ the value of Ω saturates. This effect can be understood because, at great values of γ, i.e,. strong fingering intensity in our model, the selected column *always grows* because pressure becomes great enough. For long times, then, the lateral correlation effect included in the model saturates the width. The possibility of appearance of an instability related with no saturation of the interface width is in this way forbidden.

For $\gamma>0$, there is a crossover region following the initial part of the growing. Within this region the roughness does not show a scaling behavior. The existence of the crossover region can be explained as a competition between two effects, the intercolumn interaction and the fingering process. The former tends to smooth the surface while the latter tends to increase the roughness. For lower values γ the fingering process is weak and the surface reaches its saturation width due to the intercolumn interaction effect. But, the saturation width value depends on γ because the fingering process becomes stronger as the γ coefficient increases.

The spatial scaling of the surface roughness in the stationary regime is calculated through the so-called height-height correlation function. The correlation function is defined as

$$c(r,t)=\langle |h(\mathbf{r}',t')-h(\mathbf{r}'+\mathbf{r},t'+t)|\rangle \ . \qquad (3)$$

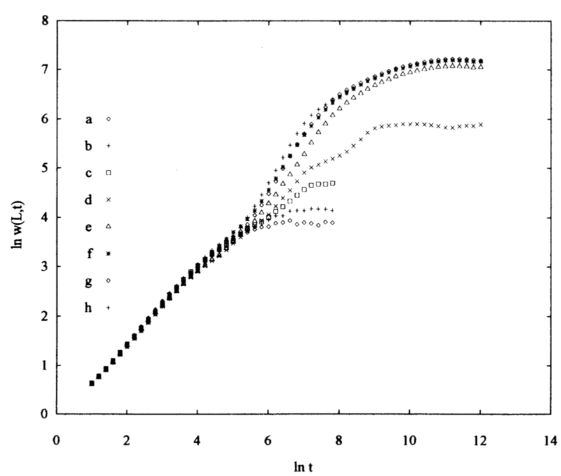

FIG. 2. Scaling of the surface width for (a) $\gamma=0$, (b) $\gamma=1\times10^{-4}$, (c) $\gamma=2\times10^{-4}$, (d) $\gamma=3\times10^{-4}$, (e) $\gamma=5\times10^{-4}$, (f) $\gamma=7\times10^{-4}$, (g) $\gamma=8\times10^{-4}$, and (h) $\gamma=10^{-3}$. The slope of the first part of the plot is $\cong0.82$.

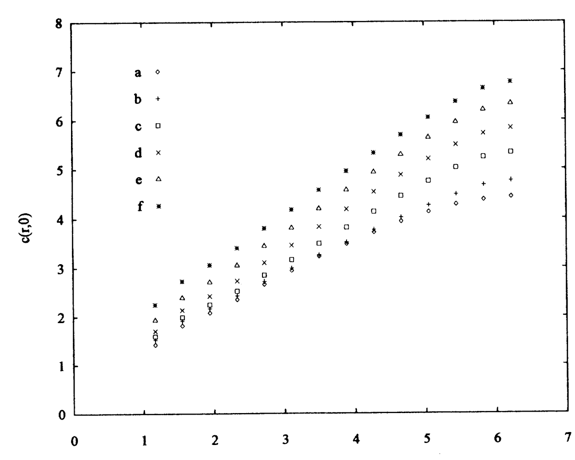

FIG. 4. Behavior of the correlation $c(r,0)$ as a function of r for different values of γ.

It is known that the correlation function behaves as [7]

$$c(r,0) \sim r^{\alpha} \quad (4)$$

for $r \ll L$, and for fixed r and short times

$$c(0,t) \sim t^{\beta} . \quad (5)$$

The behavior of the correlation function in the stationary regime is shown in Fig. 4. The values of the α exponent are the slopes of the curves as $\log(r) \rightarrow 0$. It can be seen that the value of the α exponent is dependent on the γ coefficient. For $\gamma = 0$ previous results are reproduced [2] with $\alpha \cong 0.7$. As the γ coefficient increases, the exponent α increases up to $\alpha \cong 0.95$ for $\gamma = 5 \times 10^{-4}$. This can be explained because as the fingering effect increases, the heights of the columns within a "finger" tend to be more correlated (see interface shapes in Fig. 1).

Hence, it is not possible to characterize the interface with only two exponents because one of them, the spatial scaling exponent α, depends on the degree of fingering present. The intensity of the fingering effect depends not only on the viscosity relation between the fluid involved in the experiment, but also on a number of parameters (temperature and proximity among others) which could be very difficult to quantify.

We have studied a model of a growing interface within a random porous media. The possibility of having viscous fingering effects is included through a height dependent probability for the advance of the interface. The inclusion of viscous fingering effects is regulated by a parameter. Although the temporal scaling of the interface seems not to be affected, the spatial scaling varies with the intensity of the fingering. In principle, the fingering effect could always be present in real interfaces in porous media. Nevertheless, its intensity is hard to estimate. The great variety of experimental α exponent values reported up to the present could be then a result of the influence of even small fingering effects not taken into account in the theoretical models.

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